2 First probabilists: Galileo,Cardano, Fermat, Pascal, Huygens

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(Volume I includes De ludo aleae. See Hacking for further paths.)

1 Beginnings

Although hints of probabilistic reasoning are found in Aristotle, and in the Talmud, the beginnings or modern probability date from the sixteenth century. Regular dice date from 3000 bc, so it can hardly be said that the regularity of the dice initiated the theory. The use of gambling equipment for divination, one might think, would inhibit careful calculation which might be thought to interfering in the divine domain. Yet the rise of Christianity suppressed official sanction of divination, so that this reason for lack of development of probability is also weak.

Hacking describes some early probability concepts in the Indian literature; in the Mahabarata, there is a story about Rtuparna, who estimates the number of leaves and fruit on a tree, after examining a single twig. On congratulated on being right, he says
I of dice possess the science and in numbers thus am skilled

Thus Rtuparna, one might think, is the first randomizing survey sampler.

2 Points and hazard

Two kinds of problems were first solved formally in the new theory: the first was the problem of points, the division of stakes when a game is incomplete; the second was the problem of combination of events, such as the probability of throwing a six at least once in four throws of a die. The points problem is thought to be of Arabic origin, like the algebra necessary to solve it. The Italians produced varying wrong solutions in the 16th century. Pacioli(1494):

A team plays ball in such a way that a total of 60 points is required to win the game, and each goal counts 10 points. The stakes are 10 ducats. By some incident, they cannot finish the game, and one side has 50 points and the other 20. One wants to know what share of the prize money belongs to either side.

The common Italian game of hazard, thought to be of Arabic origin, (after *al azar*, dice) used three dice, and fortunes were usually told with three dice.(It seems as if though there are many games called Hazard. There is an English two dice version that is the ancestor of Craps. There is another Hazard game that is the ancestor of Chuck-a-Luck; in Chuck-a-Luck , you bet on a particular face, three dice are thrown, and you win your stake times the number of times your face appears. Thus if you bet 1$ on 6, and it appears once you win 1$).

One of the earliest probabilistic calculations is the poem *De Vetula* ascribed to Richard de Fournival(1200-1250), that lists the number of ways three dice can fall allowing for permutations. David translates the relevant passage as follows:

If all three numbers are alike, there are six possibilities; if two are alike, and the other different there are 30 cases, because the pair can be chosen in six ways, and the other in five; and if all three are different, there are 20 ways, because 30 times 4 is 120 but each possibility arises in 6 ways. There are 56 possibilities. But if all three are alike there is only one way for each number; if two are alike and one different there are three ways; and if all are different there are six ways. The accompanying figure shows the various ways.

The accompanying table gives the correct chances of the various totals of three dice , ranging from 3 to 18, when each die falls independently with uniform probability on 1 to 6. This is the first known modern assignment of probabilities to the fall of the dice.

Galileo(1564-1642) produces the same probability assignments, but with more explicit reasoning:

The fact that in a dice-game certain numbers are more advantageous than others has a very obvious reason, i.e. that some are more easily and more frequently made than others, which depends on their being able to be made up with more variety of numbers......
since a die has six faces, and when thrown it can equally well fall on any of these, only 6 throws can be made with it, each different from all the others. But if together with the first die we also throw a second, which also has six faces, we can make 36 throws each different from all the others, since each face of the first die can be combined with each of the second,...
whence it is clear that such combinations are 6 times 6 i.e.36.
And if we add a third die... 216, each different from the others.

It seems as if Galileo was a probabilist , don't you think?
Hacking asserts that he is the first frequentist. Yet Aristotle says "the probable is that which most frequently occurs". It is hard to settle these priority claims.

Cardano(1501-1576) was a physician, a mathematician, and a gambler who published the first book in probability *De Ludo Aleae*. It did not appear until 1663, a century after it was written , c1564. He discussed both dice and knucklebones and the problem of points.
Here are two quotations from Hackings treatment:

In three casts of two dice, the number of times that at least one ace will turn up three times in a row falls far short of the whole circuit, but its turning up twice differs from equality by about 1/ 12. The argument is based upon the fact that such a succession is in conformity with a series of trials , and would be inaccurate apart from such a series.

3 Pascal and Fermat letters

There was a letter from Pascal to Fermat in 1654 asking about a problem of points. A gambler undertakes to throw a six in eight throws of a dice. Suppose he had made three throws without success, what proportion of the stake should he have on condition he gives up any further throws?

Only Fermat's reply survives.

Fermat to Pascal
1654 [undated]

Monsieur

If I undertake to make a point with a single die in eight throws, and if we agree after the money is put at stake, that I shall not cast the first throw, it is necessary by my theory that I take 1/6 of the total sum to he impartial because of the aforesaid first throw.

And if we agree after that that I shall not play the second throw, I should, for my share, take the sixth of the remainder that is 5/36 of the total.

If, after that, we agree that I shall not play the third throw, I should to recoup myself, take 1/6 of the remainder which is 25/216 of the total.

And if subsequently, we agree again that I shall not cast the fourth throw, I should take 1/6 of the remainder or 125/1296 of the total, and I agree with you that that is the value of the fourth throw supposing that one has already made the preceding plays.

But you proposed in the last example in your letter (I quote your very terms) that if I undertake to find the six in eight throws and if I have thrown three times without getting it, and if my opponent proposes that I should not play the fourth time, and if he wishes me to he justly treated, it is proper that I have 125/1296 of the entire sum of our wagers.

This, however, is not true by my theory. For in this case, the three first throws having gained nothing for the player who holds the die, the total sum thus remaining at stake, he who holds the die and who agrees to not play his fourth throw should take 1/6 as his reward.

And if he has played four throws without finding the desired point and if they agree that he shall not play the fifth time, he will, nevertheless, have 1/6 of the total for his share. Since the whole sum stays in play it not only follows from the theory, but it is indeed common sense that each throw should be of equal value.

I urge you therefore (to write me) that I may know whether we agree in the theory, as I believe (we do), or whether we differ only in its application.

I am, most heartily, etc.,

Fermat.

Pascal responded to this letter on
29 July 1654 with further discussion, and also raising a problem of the
Chevalier de Mere: compare the probability of getting a six in four throws of a die, with the probability of getting two sixes in twenty four
throws of two die. Since 4/6 = 24/36, but the probabilities are unequal, there must be an error in the Laws of Arithmetic, the Chevalier thinks.

These two letters are regarded as founding the theory of probability.

4 Huygens: The first probability text

Christianus Huygens(1629-1695), Lord of Zelem and of Zuylichem, has the honor of writing the first probability text, *De Ratiocinniis in aleae ludo,* Reasoning in Games of Chance, that gives the rules for probability calculations and defines mathematical expectation. Huygens's book appeared in 1657, a mere three years after the Fermat-Pascal correspondence.

From David(1962), Huygens' fourteen propositions run as follows :

I: To have equal chances of getting a and b is worth (a + b)/2.
II : To have equal chances of getting a, b or c is worth (a + b + c)/3.
III : To have p chances of obtaining a and q of obtaining b, chances being equal, is worth (pa + qb)/p + q.
IV: Suppose I play against an opponent as to who will win the first three games and that I have already won two and he one. I want to know what proportion of the stakes is due to me if we decide not to play the remaining games.
V: Suppose that I lack one point and my opponent three. What proportion of the stakes, etc.
VI: Suppose that I lack two points and my opponent three, etc.
VII : Suppose that I lack two points and my opponent four, etc.
VIII : Suppose now that three people play together and that the first and second lack one point each and the third two points.
IX: In order to calculate the proportion of stakes due to each of a given number of players who are each given numbers of points short, it is necessary, to begin with, to consider what is owing to each in turn in the case where each might have won the succeeding game.
X: To find how many times one may wager to throw a six with one die.
XI: To find how many times one should wager to throw 2 sixes with 2 dice.
XII : To find the number of dice with which one may wager to throw 2 sixes at the first throw.
XIII : On the hypothesis that I play a throw of 2 dice against an opponent with the rule that if the sum is 7 points I will have won but that if the sum is 10 he will have won, and that we split the stakes in equal parts if there is any other sum, find the expectation of each of us.
XIV: If another player and I throw turn and turn about with 2 dice on condition that I will have won when I have thrown 7 points and he will have won when he has thrown 6, if I let him throw first find the ratio of my chance to his.

He proves his proposition X using expectation arguments. No total probability 1, there was no such assumption.

It is certain that the gambler who wagers to throw a 6 in a single throw has one chance of winning and 5 of losing. For he has five throws against him and only one for him. Call the stakes a. He has therefore one chance of getting a and 5 chances of not getting it, so that by the second Proposition he will expect a/6. There remains 5a/6 for his Opponent. He who plays a single game of one throw therefore can only count odds 1 against 5. The proportion due to the gambler who wagers to throw a 6 in two throws, is calculated in the following way. If he throws a 6
the first time, he wins a. If he fails at the first throw, he still has another which is worth a/6 to him by the previous argument. But he has only one chance of throwing a 6 at the first throw and he has 5 chances of not getting it. He has therefore at the beginning one chance of obtaining a and 5 chances of obtaining a/6, which is worth 11.a/36 by the second proposition. There remains 25.a/36 for his opponent. He who throws and wagers for a six in 2 throws has odds therefore of 11 : 25 which is less than 1 : 2.

I like Huygens's approach, since I take expectation to be the fundamental concept, and the technically most useful approach to computations as well. After all, an expectation is just a linear functional, about which we know everything, and a set based probability is merely a horrible half complete twisted midget. No offense!