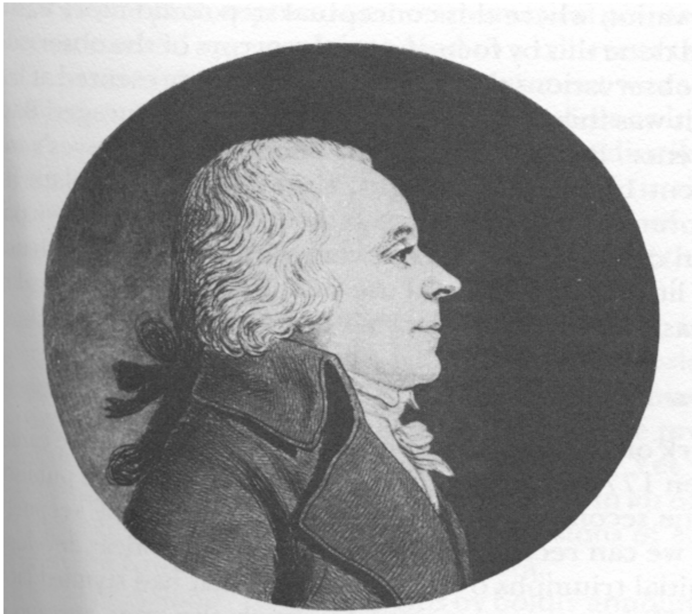
6 Laplace

0 Texts

Stigler, Stephen(1986) *The History of Statistics : The Measurement of Uncertainty Before 1900*
Laplace, Pierre Simon(1774) Mémoire sur la probabilité des causes par les évènemens. Mémoires de l'Académie royale des sciences presentés par divers savans 6:621-56. Translated in Stigler(1986).
Stigler, Stephen (1986) Laplace's 1774 memoir on inverse probability. *Statistical Science 1*
Laplace, Pierre Simon (1812) Théorie analytique des probabilités. Paris: Courcier. Reprinted as *Oeuvres completes de Laplace7, 1878-1912.* Paris: Gauthier-Villars.
Laplace, Pierre Simon (1814) Essai philosophique sur les probabilités. Paris:Courcier. Reprinted 1951 New York:Dover
Todhunter, I (1865) A history of the mathematical theory of probability New York, Chelsea (1949).
www.Britannica.com

1 Laplace the man



Pierre Simon Laplace (1749–1827), as portrayed in 1799

From the Encyclopedia Brittanica, Pierre-Simon marquis de Laplace (1749-1827) the French mathematician, astronomer, and physicist who is best known for his investigations into the stability of the solar system.

Laplace successfully applied the Newtonian theory of gravitation to the solar system by accounting for all the observed deviations of the planets from their theoretical orbits and developed a conceptual view of evolutionary change in the physical universe. He also demonstrated the usefulness of the probabilistic interpretation of scientific data.

Laplace was the son of a peasant farmer. Little is known of his early life except that he quickly showed his mathematical ability at the military academy at Beaumont. At age 18 he left his humble surroundings for Paris, determined to make his way in mathematics. He then composed a letter on principles of mechanics for the mathematician Jean d'Alembert, who recommended him to a professorship at the École Militaire.

In 1773 he began his major lifework--applying Newtonian gravitation to the entire solar system--by taking up a particularly troublesome problem: why Jupiter's orbit appeared to be continuously shrinking while Saturn's continually expanded. The mutual gravitational interactions within the solar system were so complex that mathematical solution seemed impossible; indeed, Newton had concluded that divine intervention was periodically required to preserve the system in equilibrium. Laplace announced the invariability of planetary mean motions, carrying his proof to the cubes of the eccentricities and inclinations. This discovery in 1773, the first and most important step in establishing the stability of the solar system, was the most important advance in physical astronomy since Newton. It won him associate membership in the Academy of Sciences the same year.....

In 1814 Laplace published a popular work for the general reader, Essai philosophique sur les probabilités (A Philosophical Essay on Probability). This work was the introduction to the second edition of his comprehensive and important Théorie analytique des probabilités ("Analytic Theory of Probability"), first published in 1812, in which he described many of the tools he invented for mathematically predicting the probabilities that particular events will occur in nature. He applied his theory not only to the ordinary problems of chance but also to the inquiry into the causes of phenomena, vital statistics, and future events, while emphasizing its importance for physics and astronomy.

Probably because he did not hold strong political views, he escaped imprisonment and execution during the Revolution.(...Unlike Condorcet, a contemporary student of inverse probability, who was guillotined...) Laplace was president of the Bureau des Longitudes (Board of Longitude), aided in the organization of the metric system, helped found the Society of Arcueil, a scientific society, and was created a marquis. He served for six weeks as minister of the interior under Napoleon, who thought his record as an administrator was undistinguished.

The two giants of early probability and statistics are James Bernoulli and Laplace. All the important previous probabilistic and inferential developments were captured in Laplace (1812) Théorie analytique des probabilités. I cannot find an English translation of Theorie Analytique although there is an English translation of the introduction, published separately by Laplace as Essai philosophique sur les probabilités. You can find an English description, not a translation, of the Theorie Analytique in Todhunter, 1865, which gives similar detailed description of mathematical probability from Pascal to Laplace.

Laplace’s Definition of Probability (From Essai Philosophique)

The theory of chance consists in reducing all events of the same kind to a certain number of cases equally possible, that is to say, such as we may be equally undecided about in regard to their existence, and in determining the number of cases favorable to the event whose probability is sought. The ratio to that of all the cases possible is the measure of this probability, which is thus simply the fraction whose numerator is the number of favorable cases, and whose denominator is the number of all case possible.

2 Laplace and inverse probability

Laplace's first work on probability, which begins with first general statement of Bayes theorem, appeared in *Mémoire sur la probabilité des causes par les évènemens* This paper is translated in Stigler,1986. Stigler is the best source on Laplace, as on most of the other writers in this period. Whereas Bayes applied inverse probability to the binomial, Laplace applied it generally, and in particular to inference from astronomical data.

Laplace's opening principle(1774,p623):

If an event can be produced by a number n of different causes, then the probabilities of these causes given the event are to each other as the probabilities of the event given the causes, and the probability of the existence of each of these is equal to the probability of the event given the cause, divided by the sum of all the probabilities of the event given each of these causes.

I am sure scholars want to see the French original:

Si un évènement peut être produit par un nombre n de causes différentes, les probabilités de l'existence de ces causes prises de évènement , sont entre elses comes les probabilités de l'évènement prises de ces causes, et la probabilité de l'existence de chacune d'elles, est égale á la probabilité de l'évènement prise de cette cause, diviseé par la somme de toutes les probabilités de l'évènement prises de chacune de ces causes.

Laplace's statement of inverse probability is correct only if the probabilities of causes are a priori equal; Laplace is not explicit about these prior probabilities until the Theorie. Laplace, according to Stigler, was not aware of Bayes's work until about 1780, and his statement of inverse probability is certainly in quite a different form to Bayes. Bayes deserves and has priority because he made the earlier statement, and is explicit and concerned about the prior probabilities.

3 Application to determining the constitution of balls in an urn

(from Laplace 1774)

Problem 1

Suppose that an urn contains an infinite number of white tickets and black tickets in an unknown ratio; p+q tickets are drawn, of which p are white and q are black: required the probability of drawing m white tickets and n black tickets in the next m+n drawings.

Laplace assumes implicitly that the unknown ratio in the urn is uniformly distributed a priori.

In discussing this problem, Laplace give the first statement of consistency of posterior distributions:

La solution de ce Probléme donne une méthode directe pour déterminér la probabilité des évènemens futurs d'aprés ceux qui sont déja arrivés; je mais cette matiére étant fort étendue, je me bornerai ici á donner une démonstration assez singuliére du théorème suivant.

On peut supposer les nombres p et q tellement grands, qu'il devienne aussi approchant que l'on voudra de la certitude, que le rapport du nombre de billets blancs au nombre total des billets renfermés dans l'urne, est compris entre les deux limits $\frac{p}{p+q}-\omega, \frac{p}{p+q}+\omega$ *, ω pouvant être supposé moindre qu'aucune grandeur donné.*

4 An inverse Problem of points:

Next, Laplace considers inverse probability applied to a problem of points. I am glad these problems went out of style in another 100 years or so. This seems a routine application.

Let x denote the skill of the player A, and 1- x the skill of the player B; suppose that A wants f games in order to win the match, and that B wants h games: then, if they agree to leave off and divide the stakes, the share of B will be a certain quantity which we may denote by φ(x,f,h). Suppose the skill of each player unknown; let n be the whole number of games which A or B ought to win in order to entitle him to the stake.

Then the share of B is $\int_0^1 x^{n-f}(1-x)^{n-h}\varphi(x,f,h)dx \,/ \int_0^1 x^{n-f}(1-x)^{n-h}dx$

5 Combination of observations

Problem 3 *Dèterminer le milieu que l'on doit prendre entre trois observations donnès d'un même phènoméne.*

Laplace's milieu is actually the median, which Laplace showed minimized the expected absolute error.

From Stigler, p106

Laplace's discussion was conducted in reference to three figures (see Figure 3.1). Laplace considered the problem to be as follows: Given three observed times of a phenomenon (a, b, and c) along the time axis AB, find the point V that we should take as the true time of the phenomenon. In determining this point he supposed that if V is the true instant of the phenomenon, then the probability of an observation differing from the truth by an amount x was given by a curve y = φ(x), shown as ORM in the middle diagram. Two points remained to be settled. What curve should be taken as the error curve φ(x) And given φ(x), how should the mean be determined?
Laplace listed three conditions that φ(x) should satisfy: First, the curve should be symmetrical about V "because it is just as probable that the observation deviates from the truth to the right as to the left. ' Second, the curve must decrease toward the axis KP "because the probability that the observation differs from the truth by an infinite distance is evidently zero." Third, the area under the curve must be one "because it is certain that the observation will fall on one of the points of the line KP." These conditions did not determine φ(x), but they did permit some general reflections on the problem… As Laplace wrote, "But of an infinite number of possible functions, which choice is to be preferred?"

Laplace argued that there was "no reason to suppose a different law for the ordinates than for their differences", and this leads to the Laplace or double exponential error distribution $\varphi(x) = \exp(-m\,|\,x\,|)/2m$.

To solve the problem of finding the posterior median of V|a,b,c, Laplace first computes $\varphi(V \mid a,b,c,m) = \dfrac{\varphi(a-V)\varphi(b-V)\varphi(c-V)}{\int \varphi(a-V)\varphi(b-V)\varphi(c-V)dV}$ ***which depends on the unknown scale parameter m. Laplace finds the posterior distribution of m given a,b,c,*** $\varphi(m \mid a,b,c)$.
Then he computes the marginal posterior integrating out m,

$$\varphi(V \mid a,b,c) = \int \varphi(V \mid a,b,c,m)\varphi(m \mid a,b,c)dm$$

and looks for the median of this posterior distribution as "the mean between the three observations".

This is what he intends to do, although Stigler points out that his actual computation was slightly different. We see that Laplace is using probability in a consistent way to handle all the quantities he is uncertain about, a,b,c,V,m, that is carrying out the characteristic Bayesian analysis of uncertainty.

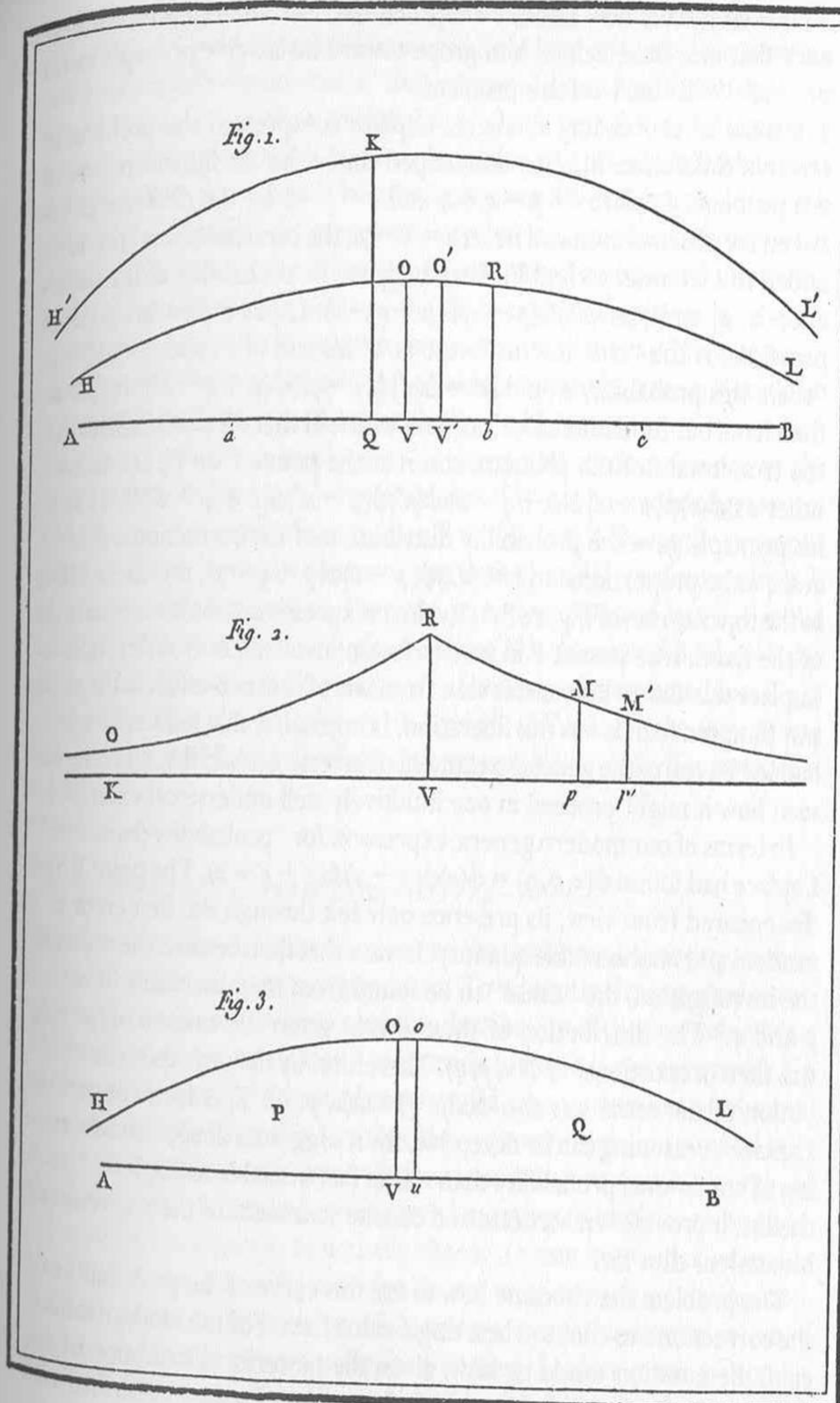


Figure 3.1. Diagrams accompanying Laplace's 1774 memoir on inverse probability. His Figure 2 shows the double exponential density, and his Figures 1 and 3 show posterior distributions, the latter illustrating his argument that the posterior median minimizes the posterior expected error. (From Laplace, 1774, facing p. 656.)